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Comment on "Nucleon elastic form factors and local duality"

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We comment on the papers "Nucleon elastic form factors and local duality" [Phys. Rev. **D62**, 073008 (2000)] and "Experimental verification of quark-hadron duality" [Phys. Rev. Lett. **85**, 1186 (2000)]. Our main comment is that the reconstruction of the proton magnetic form factor, claimed to be obtained from the inelastic scaling curve thanks to parton-hadron local duality, is affected by an artifact.

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Recently an inclusive electron-proton scattering experiment [1] has been performed at the Jefferson Lab ($JLab$) in the resonance production region for values of the squared four-momentum transfer Q^2 between ~ 0.45 and ~ 3.3 $(GeV/c)^2$. The aim was to investigate the connection among the resonance and the scaling regions, known as parton-hadron local duality [2]. The new data were found to exhibit the local duality for each of the most prominent proton resonances. In [1] a fit to the average strength of all the resonances was carried out and, thanks to the parton-hadron local duality, interpreted as the *scaling* curve. Here below, we refer to such a fit as the $JLab$ fit, viz.

$$F_2^{(JLab)}(\xi) = \xi^{0.870}(1-\xi)^{0.006} \cdot [0.005- \\ 0.058(1-\xi) - 0.017(1-\xi)^2 + \\ 2.469(1-\xi)^3 - 0.240(1-\xi)^4] \qquad (1)$$

where $\xi \equiv 2x/[1+\sqrt{1+4M^2x^2/Q^2}]$ is the Nachtmann variable, which includes the effects of target-mass corrections, improving at finite Q^2 the Bjorken scaling variable x. In order to constrain the large-ξ behavior of the $JLab$ fit the authors of [1] have employed $SLAC$ data up to $Q^2 = 8$ $(GeV/c)^2$. This means that the highest ξ-point constraining the $JLab$ fit is $\xi \simeq 0.86$, corresponding to the $\Delta(1232)$ location at $Q^2 = 8$ $(GeV/c)^2$.

An interesting question is whether local duality may be applied to the proton elastic peak [2–5]. If local duality holds also in the unphysical region extending up to $\xi = 1$ (which corresponds at finite Q^2 to $x > 1$), the proton magnetic form factor $G_M^p(Q^2)$ can be obtained from the moment of order n of the scaling function, $F_2^p(\xi)$, viz. (cf. [3])

$$G_M^p(Q^2) = \sqrt{\frac{2-\xi_{el}}{\xi_{el}^n}\mu_p^2 \frac{1+\tau}{1+\mu_p^2\tau}\int_{\xi_\pi}^{\xi^*} d\xi \xi^{n-2} F_2^p(\xi)} \quad (2)$$

where μ_p is the proton magnetic moment, $\tau = Q^2/4M^2$, $\xi_{el} = 2/[1+\sqrt{1+1/\tau}]$, $\xi^* = min[1, Q/M]$ and ξ_π is the pion production threshold. Note that $\xi_\pi(\xi_{el}) = 0.41(0.50), 0.63(0.70), 0.76(0.81), 0.83(0.87), 0.90(0.92)$ at $Q^2 = 0.45$, 1.4, 3.0, 5.0, 10 $(GeV/c)^2$, respectively. In [6], adopting for $F_2^p(\xi)$ the $JLab$ fit (1) and considering only $n = 2$, the reconstructed $G_M^p(Q^2)$ was shown to agree with the data within 30% up to $Q^2 \sim 7$ $(GeV/c)^2$. This result is at variance with the findings of Refs. [2–5].

We start noting that in the righthand side of Eq. (1) the term proportional to $(1-\xi)$, which any way is not consistent with quark counting rules, has a negative coefficient, so that $F_2^{(JLab)}(\xi)$ may be a monotonic increasing function at large ξ. This is indeed the case as shown in Fig. 1 by the dashed line[1], which exhibits an anomalous behavior at $\xi \gtrsim 0.9$, i.e. beyond the highest ξ-point constraining the $JLab$ fit of [1].

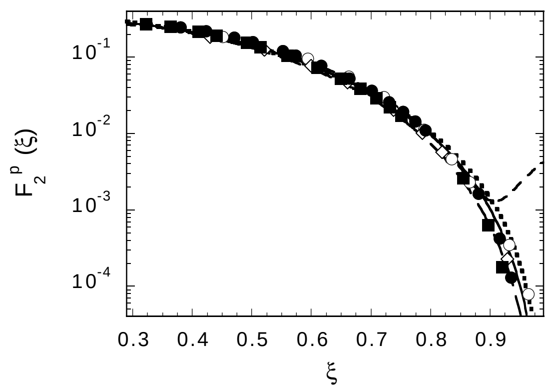

FIG. 1. The proton scaling function $F_2^p(\xi)$ versus the Nachtmann variable ξ. The dashed and solid lines correspond to the $JLab$ fit (1) of [1] and to our modified $JLab$ fit given by Eq. (3), respectively. The open dots and diamonds, and the full dots and squares are the average strengths, obtained using the $SLAC$ parameterization [7] of the proton structure function, in the $\Delta(1232)$, $S_{11}(1535)$, $F_{15}(1680)$ and "higher-mass" resonance regions, as defined in [1], respectively. The long-dashed and dotted lines correspond respectively to the GRV set [8] of PDF's and to the NMC parameterization [9], omitting for the latter its $1/Q^2$ term (see text), evaluated at $Q^2 = 10$ $(GeV/c)^2$.

In order to clarify the impact of the anomalous shape of the $JLab$ fit on the reconstruction of $G_M^p(Q^2)$

[1]Note that in Eq. (1) the term $(1-\xi)^{0.006}$ ensures that $F_2^{(JLab)}(\xi = 1) = 0$, but in practice it has no effect at all for ξ up to 0.9999, as it can be easily checked numerically.

through Eq. (2), we have simply developed a *modified* version of the *JLab* fit, which coincides with the original one within ±10% for $\xi \lesssim 0.86$, but exhibits a monotonic decreasing behavior at larger ξ, viz.

$$\tilde{F}_2^{(JLab)}(\xi) = \xi^{0.940} \left[2.650(1-\xi)^{3.38} + 0.240(1-\xi)^4\right] \quad (3)$$

In Fig. 1 the modified *JLab* fit is reported as the solid line and compared with the average strengths of the most prominent proton resonances, generated using the parameterization of the inelastic *SLAC* data of [7]. It can be seen that our modified *JLab* fit is in reasonable agreement with the *SLAC* resonance averages up to very large values of ξ. Finally, the proton structure function $F_2^p(\xi)$ evaluated at $Q^2 = 10$ $(GeV/c)^2$ using the *GRV* set [8] of parton distribution functions (*PDF*'s) and the *NMC* parameterization of [9] is shown in Fig. 1. Note that the *NMC* fit contains an explicit power correction term proportional to $1/Q^2$, which has been excluded. Indeed, as shown in [3], the replacement of the Bjorken variable x with the Nachtmann variable ξ is an approximate way to consider target mass (*TM*) corrections at large Q^2. Therefore, the inclusion of the $1/Q^2$ term of the *NMC* fit (which incorporates already *TM* effects) and, at the same time, the use of the variable ξ would lead to an overcounting of *TM* effects. From Fig. 1 it can be seen that up to $\xi \simeq 0.85$ the shape of the *JLAB* fit is not inconsistent with standard *PDF* expectations as well as with the *NMC* fit, provided *TM* effects are properly included. This is at variance with the results reported in Fig. 2 of [1]. There, however, *TM* effects were not included consistently in the curves labeled "MRS(G)" and "CTEQ4". We have checked that, after proper inclusion of *TM* corrections, those curves become close to the results labeled "NMC" (cf. also Fig. 3 of [3]).

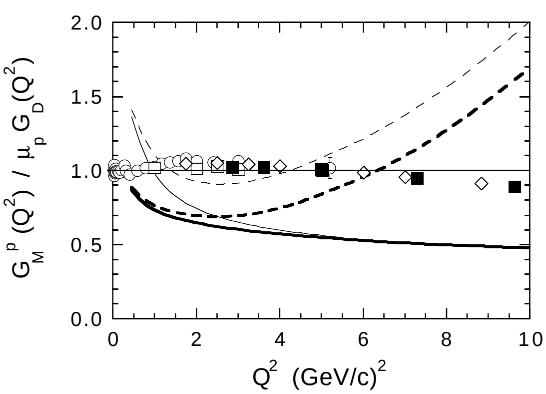

FIG. 2. The proton magnetic form factor $G_M^p(Q^2)$, divided by its dipole expectation $\mu_p G_D(Q^2) \equiv 2.793/(1+Q^2/0.71)^2$, versus Q^2. Open dots, squares, diamonds and full squares are the experimental data from Ref. [10](a), (b), (c) and (d), respectively. The dashed and solid lines are the results of Eq. (2) obtained using the *JLab* fit (1) and its modified version (3), respectively. Thick and thin lines correspond in Eq. (2) to $n = 2$ and $n = 10$, respectively.

In Fig. 2 the proton magnetic form factor $G_M^p(Q^2)$, resulting from the application of the parton-hadron local duality [Eq. (2)] using the original and our modified *JLab* fits, is shown and compared with the data. In evaluating Eq. (2) we have considered both $n = 2$ and $n = 10$. The former case is the only one employed in [6], while the latter is representative of the case of higher moments which are more sensitive to the shape of the scaling curve at large ξ. It can be seen that : i) the results we have obtained using the *JLab* fit (1) coincide with those of [6] for $n = 2$ (thick dashed line), but exhibit a remarkable dependence on the order n of the moment (compare thin and thick dashed lines); ii) the anomalous shape of the *JLab* fit (1) heavily affects the reconstruction of $G_M^p(Q^2)$ for $Q^2 \gtrsim 5$ $(GeV/c)^2$ (compare thick dashed and solid lines); iii) using our modified *JLab* fit the values of $G_M^p(Q^2)$ obtained via the application of parton-hadron local duality, underestimates the data by a factor of $\simeq 2$ for $Q^2 \gtrsim 2$ $(GeV/c)^2$ (see thick solid line); iv) for $Q^2 \lesssim 2$ $(GeV/c)^2$ the reconstructed $G_M^p(Q^2)$ appears to be close to the experimental data only if $n = 2$ is adopted (compare thin and thick lines).

To sum up, the main conclusion of Ref. [6], concerning the possibility of reconstructing the proton magnetic form factor from the inelastic scaling curve, is the result of an artifact in the *JLab* fit (1) of Ref. [1]. Using our modified *JLab* fit [see Eq. (3)] we have shown that the application of the parton-hadron local duality, as given by Eq. (2), fails to reproduce existing data on $G_M^p(Q^2)$ at least for Q^2 up to ~ 10 $(GeV/c)^2$, in agreement with the findings of Refs. [2–5].

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